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The Structural Relationships between Conceptual and Procedural Knowledge in Differential Equations

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Abstract

The aim of this study was to investigate the relationship between conceptual and procedural knowledge of the Second Order Linear Homogeneous Differential Equations based on a structural model. Students majoring in Science and Engineering at the Shahid Rajaee Teacher Training University in 2009-2010 were considered as a statistical population. The primary test consisted of 30 multiple-choice questions based on the goal-content's table of the subject. Thirty students from Science and Engineering department were chosen in a pilot study by the method of cluster sampling. After analyzing data, 15 questions were eliminated due to lack of consistency with other questions and also due to their difficulty and discrimination indices. Finally, a multiple-choice test with 15 questions was designed. Its reliability was satisfactory ($\alpha = 0.79$). One hundred and twenty two students were selected by cluster sampling method. The experimental model of learner's knowledge was compiled and by the use of structural equation modeling the direct and total effects of factors determined. Analysis of the final data showed that there is a meaningful direct effect on knowledge of 'concepts' with knowledge of 'modeling' ($p<0.01$); knowledge of 'algorithm' with knowledge of 'application' ($p<0.01$); knowledge of 'application' with knowledge of 'algorithm' ($p<0.01$); and knowledge of 'application' with knowledge of 'theorems' ($p<0.05$). These results demonstrate that understanding concepts, algorithm, applications, and mathematics modeling has a positive effect on the relationship between conceptual and procedural knowledge. Therefore, the outcome of challengeable tasks in the process of teaching could lead to students' learning achievement in mathematics.

Keywords: Conceptual Knowledge, Procedural Knowledge, Structural Relationships

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1. Introduction

Procedural knowledge refers to skills required for doing mathematics assignments and solving mathematical problems. This knowledge includes knowing notions used for representing some mathematical concepts and a set of laws, formulas and procedures, for solving mathematical problems. The key feature of these procedures is that they are linear and predetermined series. On the contrary, conceptual knowledge is the knowledge of mathematical realities, features and relations in a network of different information and strong relationships (Hiebert and Lefevre, 1986, cited in Kwoen, 2004).

Haapasalo and Kadigevich (2000) define conceptual knowledge as knowing about and moving skillfully in a network of concepts, laws (algorithms and procedures) and even solved problems which may introduce a new concept or law. While procedural knowledge is defined as dynamic use of laws, algorithms or certain procedures, in related frames, procedural knowledge entails automatic and unconscious phases. However, conceptual knowledge is usually based on conscious thinking. Haapasalo and Kadigevich (2003) have articulated the results of some studies about conceptual and procedural knowledge, as follows:

1. Inactivation view: conceptual and procedural knowledge are not related (Resnick and Omanson, 1987).
2. Simultaneous activation view: procedural knowledge is the necessary and sufficient condition for conceptual knowledge (Haapasalo, 2003).
3. Dynamic interaction view: conceptual knowledge is the necessary, but not sufficient condition for procedural knowledge (Byrnes and Wasik, 1991).
4. Genetic view: procedural knowledge is the necessary, but not sufficient condition for conceptual knowledge (Gray and Tall, 1994).

Different educational approaches and strategies, students' different aptitudes and different subjects and problems in mathematics help one claim that none of the four above- mentioned statements can give a conclusive conclusion about the relationship between these two types of knowledge. Based on different research findings, researchers concluded that there is an interactive and reciprocal relationship between these two types of knowledge, in such a way that improvement in one, causes improvement in the other and vice versa (Rittle – Johnson and Siegler 1998).

The relationship between the conceptual and the procedural knowledge is an important subject that has attracted the attention of many researchers – especially mathematics education researchers. Ayden and Ubuz (2010) consider the relationship between these two types of knowledge in primary school mathematics in these subjects: counting (Gelman, Meck and Merkin, 1986), summing one digit numbers (Baroody and Gannon), summing multiple digit numbers (Hibert and Wearne, 1996), fractions (Rittle–Johnson and Seigler, 1998), decimal fractions (Resnick et. al, 1989) and percentage (Lembke and Reys, 1994). Ayden and Ubuz also mention research on solving linear equations in high school and university (Star et. al, 2005), calculating (Engelbrecht and Hardiny, 2005), mathematics equivalents (Knuth and Stephens, 2006) and algebra and analytic geometry (Webb, 1979)

Schneider and Stern (2010) carried out a study with 230 fifth and sixth graders on their conceptual and procedural knowledge .Using structural equation modeling; they developed a lesson in which there was a mutual and direct relationship between concepts and procedures. They found that previous conceptual knowledge of the students who are new to mathematics can be used for developing a new procedural knowledge.

Afamasaga (2004) investigates the effect of concept map and Vee diagram on conceptual perception of differential equations on a student. The first concept maps which were drawn by this student were imperfect and limited: many concepts and the relationships between them were ignored. However, gradually, during the research process, the concept maps drawn for each subject showed that this method has a positive effect on conceptual perception. Moreover, the final Vee diagram showed that this student would first focus on the problem, using related theorems and concepts, then would specify the equation type. Then based on the learned algorithms, he would choose the most proper equations for solving differential equations. Having reached the final answer, he would use geometrical diagrams to make sure that his answers were true and eventually, he would compare the results with the results gained from the theorems. In fact, the student would use his procedural knowledge to enhance his conceptual knowledge and established a relationship between the two types of knowledge by returning to procedural knowledge.

Waddy, Kim and Glass (2009) designed a tutoring session based on the framework given by Rasmussen (2001), in which an electronic dialogue on all instructional goals of the lesson was established between the instructor and the student. Nine students majoring Science and Engineering who had registered in differential equations course and one of the experienced instructors participated in this study. In the electronic dialogue, first the instructor would ask the student to tell him what kind of answer (number, variable …) he is after to solve the problem. By so doing, the student, knowing different theorems and concepts, would recognize the overall form of the equation. Then the students should find the suitable algorithm from among many algorithms to solve the problem and use this procedure to solve the equation. While solving the equation, he would mark the key steps. To ensure that the final answer was true and logical, it was compared with other theorems' results. At the end, the student was asked to draw the geometrical diagram of the equation's answer. In this research, the student became familiar with different differential equations and the relationship between them, and different differential equations answers. Moreover, the appropriacy of the strategies employed and the methods of establishing the accuracy of the solutions were taught. So, starting from procedural knowledge and developing conceptual knowledge, the student would gain a deep understanding of differential equations.

Based on the reported research findings, it can be said that these two types of knowledge are not learned independently rather there is a tandem relationship between them. In this process, first procedural knowledge is gained, and this brings about the conceptual knowledge; and sometimes a reverse process occurs. The learner's characteristics and previous knowledge, the subject, and learning theories which help teaching, are effective factors in this process (Haapasalo, 2003).

The learner's cognitive construct is a set of information, concepts, principles, and organized generalizations which s/he has learned in one of the (science) fields. This cognitive construct is like a pyramid. General problems and concepts are on the top of the pyramid and detailed information and knowledge of apparent realities are at the base of the pyramid. So, all topics are more general and abstract than the topics which are below that (Ivie, 1998). Meaningful learning happens when the learner has learned previously the concepts and information required for learning new topics and can establish a good relationship between new concepts and required concepts. Ausubel (1963) has called this process subsumption, in which the new and previous concepts change, in a way that the general concept range which is in the learner's cognitive construct is widened and entails the new concept. Furthermore, the new concept gradually becomes like the cognitive construct which is attracted to, and applies its characteristics (Cardellini, 2004, Kadivar, 2007).

According to "meaningful learning" theory, teaching materials should be programmed in such a way that, first the most general and abstract concepts and thoughts are be introduced generally and briefly; then, the secondary and detailed topics gradually be introduced. This educational approach applies to natural phase of cognitive construct formation (Sigler and Saam, 2006).

For teaching each concept, at upper levels of learning, in teaching any concepts, attention should be paid first to the relationship between that concept and the previous and next concepts. So, the learners learn these topics not as separated parts, but like a polished integrated network. Perkins (1995) believes that every person should have control on the inner relationship between different constituent factors of each type of knowledge to become a skillful learner and to use the acquired knowledge and concepts properly in different situations (cited in Corte, Lieven, and Chris, 2004).

Based on what is said before, by studying different differential equation books, this research intends to examine the conceptual and procedural knowledge of second order linear homogenous differential equation and examine their relationship with the help of structural equation modeling. Since no experimental model for teaching and learning second order differential equations was found, the theoretical model of this research is formulated on the bases of the framework which is utilized in different differential equations books and on the views of the instructors who have more than ten years teaching experience. In this research, procedural knowledge is the knowledge of theorems, concepts and algorithms for solving second order linear homogenous differential equations, and conceptual knowledge is the modeling of physics' problems with the help of second order linear homogenous differential equations and using first order differential equations for solving some of the second order differential equations.

Figure 1 shows the concepts of second order linear homogenous differential equations and their relationships in the form of a theoretical model.

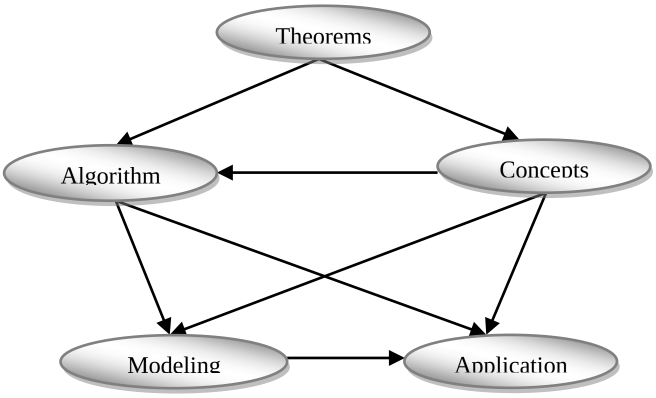

Figure1. Theoretical model of second order linear homogenous differential equations

The first topic which is set forth in most of differential equation books (in second order differential equation sections) are theorems which argue about these equations' characteristics. These theorems are: existence and uniqueness theorem with initial conditions, convolution principal axiom of function theorem, wornskin of answer theorem, general answers of an equation theorem and reduction of order theorem (Ramezani, et.al 2007). Experienced instructors belive that since this section's theorems are related to second order differential equations' answers, it is necessary to teach these theorems before arguing methods of solving these kinds of equations. So, the first factor of theoretical model in this research is "theorems".

The next topic in this section is the concepts of general form of different second order differential equations and their classification. According to different concepts given in different differential equation books, these equations are linear and non- linear equations. In most of these books, only linear equations are discussed. They are introduced as homogenous and heterogeneous equations. Homogenous equations are divided into two groups: constant coefficient and variable coefficient equations (Nikoukar, 2002), so the second effective factor in this model is called "concepts".

The third factor of the theoretical model of this research is solving homogenous equations with constant coefficient and variable coefficient (such as Cauchy – Euler's differential equation, and Legendre) which can be changed to constant coefficient equations (Boyce and Diprima, 2008). These equations are called “algorithm” because of the similarity they have in their solving algorithm. Experienced instructors believe that paying attention to the theorems’ results and knowing general form of second order differential equations are necessary for true using of these algorithms. So, the two factors of the theorems and concepts have an important role in the algorithm component formation.

One of the reasons of differential equations importance is functional role of these equations in modeling physical phenomenon such as electrical circuits and mechanical vibrations (related to bob-coil systems). This concept is discussed in some books on differential equation, (Shidfar, 1999). So the next factor in this section is related to modeling physical phenomenon which is under the influence of the two factors (concepts and algorithm).

The last factor in this model is to solve special homogenous equations with variable coefficients. These questions are differential equations without related variable, equations without independent variables and complete equations whose answer is gained by changing these equations to a first order differential equations (Ramezani et. al, 2007). This factor is called “application” of equations.

So, the research questions in second order homogenous differential equation are:

- Does knowledge of concepts influence knowledge of modeling?
- Does knowledge of algorithm influence knowledge of application?
- Does knowledge of application influence knowledge of algorithm?
- Does knowledge of application influences knowledge of theorems?

2. Methodology

Population, sample and test preparation procedures

Since this research is done for improving teaching, and giving solutions for enhancing the quality of teaching, it is a kind of pure research. The method used is correlation, in which the relationship between different variables is discovered and distinguished by correlation coefficient. So, first the influential factors are distinguished, and then their structural relationship is examined.

Statistical population of this research is all undergraduate students of Science and Engineering of Shahid Rajaee Teacher Training University (SRTTU), Tehran, in the second semester of 1389- 1390. First, based on theoretical understanding and previous research, and interviewing three instructors of mathematics who have been teaching differential equation for more than ten years at SRTTU, all the content of second order homogenous differential equations were divided into three parts: becoming familiar with second order homogenous differential equations, solving second order homogenous differential equations with constant coefficient and solving second order homogenous differential equation with variable coefficients. Then a test with thirty multiple choice questions was designed based on goal – content's table of this section. To ensure the validity of the test, teaching objectives of this lesson were determined by studying previous research and interviewing instructors. Then, for each objective a component was determined and two or three items were designed for the component. These items from the prototype test. The next step was the confirmation of the goal – content table (as to "do these questions cover the goals?") by experienced instructors. To estimate the face validity and confirming that the listed questions and items are suitable for testing the goals under consideration, or not "we enjoyed the help of three connoisseur instructors. The test items were revised by these instructors. After making required changes and corrections, the test was put in a pilot study.

By method of cluster sampling, thirty students of Science and Engineering were chosen as the sample of the primary test. Data analysis proved that fifteen items were not statistically sound and were deleted, because of lack of internal consistency with other test items and not having satisfactory difficulty and discrimination indices. At the end, a test of fifteen multiple questions covering all teaching goals of course was developed. The test reliability turned out to be satisfactory ($\alpha = 0.79$). Furthermore, test items were examined in terms of discrimination and difficulty indices. Discrimination index showed that the items have internal correlation at the level of 95%. The difficulty index of the items was between 30% and 70 %, which is statistically satisfactory (Allen and Yen, 2001).

Analyzing questions by Cronbach Alpha method of internal consistency of items, in addition to determining homogeneity and compatibility of each item with all items, by deleting heterogeneous items and correcting the test, the reliability of the whole exam improved (Sarmad, Bazargan and Hejazi, 2008). In examining items by Cronbach method the reliability of the whole exam is 0.789 and is considered as reliability index.

Deleting items it became clear that the reliability of other items decreases. This shows the homogeneity of items.

It seemed necessary to determine the sample size before doing path analysis. Since there is no general method or formula for determining the size of the sample and usually for each variable between 5 to 10 sample and, in sum, at least 100 samples is recommended Sarmad, Bazagan and Hejazi, 2008). Based on the number of behavioral objectives and 15 multiple – choice items, through cluster sampling, 122 students were asked to participate in the present study to collect data for discovering experimental learning model.

Data analysis: Since the method of this research is correlation, structural equation modeling is used for analyzing the data. This method is used for examining and testing structural relationships of a group of observed and latent variables in order to be able to examine a set of hypothesis in the form of a model (Nusair and Hua, 2010).

For discovering and developing structural relationships of second order homogenous differential equations, first correlation matrice of all constituent concepts of the topic under consideration was set. Then required calculations were done, using path analysis. The experimental model of the research was designed, based on quantification and finally its goodness of fit was examined by fitting indexes.

When a correlation matrice which has come from the sample under examination is defined and determined by a group of regression equations, the model can be analyzed by related software and its fitness for the population – out of which the sample is drawn- can be examined (Sarmad, Bazagan and Hejazi, 2008). So, correlation matrice of the previous model used in Table 1 is considered as primary data for analyzing structural equation model.

Table 1. The correlation matrices of the factors

factor	theorems	concepts	algorithm	modeling	application
theorems	–	–	–	–	0.41
concepts	0.41	–	–	–	-
algorithm	0.57	0.23	–	–	0.23
modeling	–	0.25	0.44	–	-
application	–	0.40	0.30	0.34	-

According to the table above, it can be said that the knowledge of theorems and the knowledge of algorithm of second order homogenous differential equations have the highest correlation index, $p < 0.01$ and the knowledge of algorithm has a satisfactory correlation with the knowledge of modeling. But, knowledge of concepts and knowledge of application has no significant correlation with the knowledge of algorithm ($p < 0.05$).

By the use of correlation matrix and path analysis, the relationship between the concepts is shown as what is in Figure 2. In this figure, latent variables (concepts) are in oval shape and observed variables (15 items of the test) are in square shapes. The relationship between the concepts is shown with one – way lines and the numbers on the lines show the rate of standard direct relationships between the concepts.

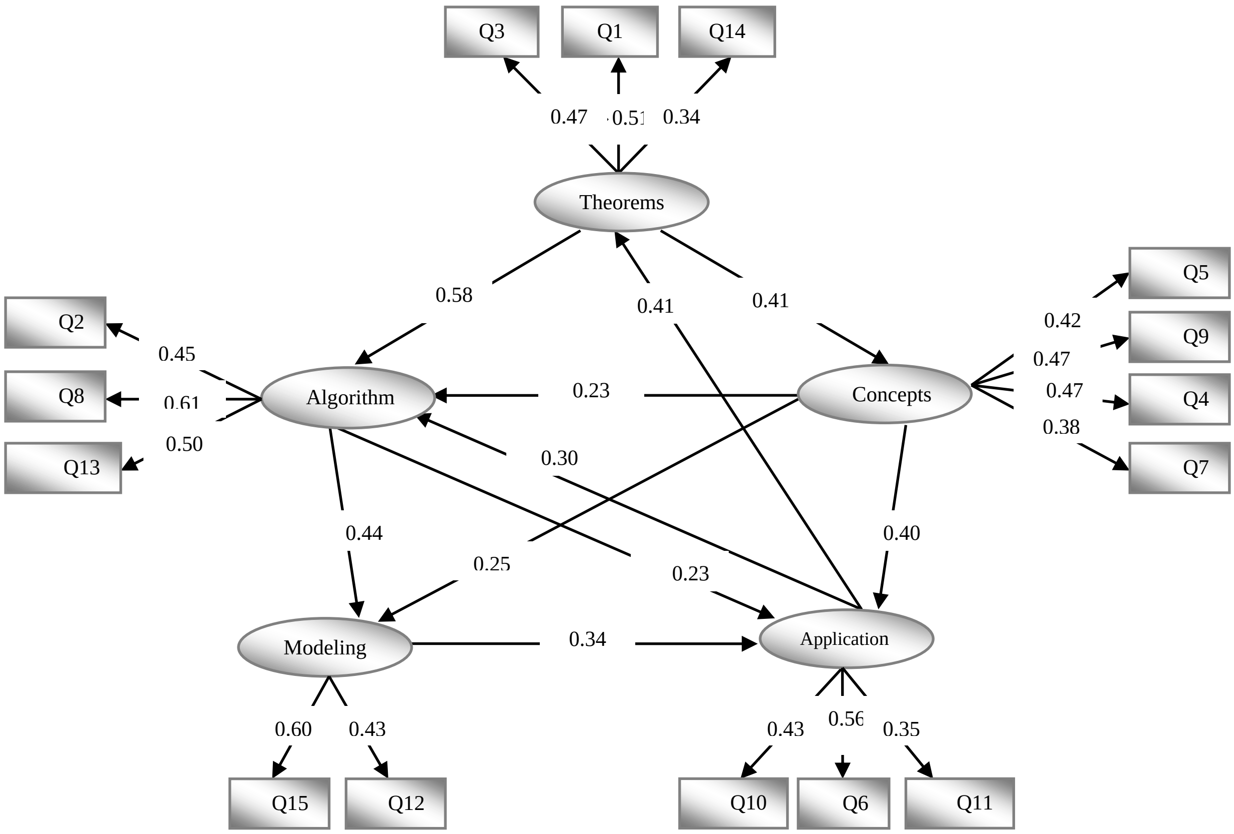

Figure2. Estimated structural model

As it is seen in Figure 2, concepts and application factors are remained in the model, because of having an indirect relationship with at least one of the intermediary factors of algorithm. The quantity index of direct relationships between factors in the final model is shown in Table2.

Table 2. The direct effect of factors on the model

The direct effect of factors on the model	estimated quantity	standard error	standardized quantity (β)	t index	significance level
theorems with concepts	0.473	0.308	0.408	1.563	P>0.05
theorems with algorithm	0.908	0.499	0.567	1.820	P>0.05
concepts with algorithm	0.313	0.322	0.226	0.972	P>0.05
algorithm with modeling	0.372	0.216	0.438	1.722	P>0.05
concepts with modeling	0.243	0.084	0.246	2.893	P < 0.01
modeling with application	0.266	0.187	0.341	1.422	P>0.05
concepts with application	0.365	0.238	0.398	1.534	P>0.05
algorithm with application	0.288	0.082	0.299	3.512	P<0.01
application with algorithm	0.182	0.066	0.231	2.758	P<0.01
application with theorems	1.121	0.533	0.413	2.103	P<0.05

As it is seen in the table above, the knowledge of concepts has a direct significant relationship with the knowledge of modeling. The direct relationship between the algorithm knowledge and the application knowledge is significant and there is a significant recurrent relationship between these two factors. Knowledge of application has also a direct significant relationship with the knowledge of the theorems.

According to standard quantity, the total effect (direct and indirect) is seen between the factors. Except for the total effect between the knowledge of concepts and the knowledge of algorithm, other effects are significant (Table 3).

Table 3. The total effect of factors on the model

The direct effect of factors on the model	estimated quantity	standard error	standardized quantity (β)	t index	significance level
theorems with concepts	0.605	o.308	0.408	1.964	P<0.05
theorems with algorithm	1.056	0.499	0.659	2.116	P<0.05
concepts with algorithm	0.336	0.322	0.226	1.043	P>0.05
algorithm with modeling	0.520	0.216	0.438	2.407	P>0.05
concepts with modeling	0.474	0.084	0.246	5.643	P < 0.01
modeling with application	0.440	0.187	0.341	2.353	P<0.05
concepts with application	0.654	0.238	0.432	2.748	P<0.01
algorithm with application	0.387	0.082	0.299	4.720	P<0.01
application with algorithm	0.182	0.066	0.231	2.857	P<0.01

application with theorems	1.087	0.533	0.401	2.039	P<0.05

There are different indexes for examining model fitness. In this research, fitness index of chi square χ^2= 62.021, with the degree of freedom of (df= 78) and significant level of p= 0.907 is used. This index shows the model fitness. Goodness fit index and root mean square error of approximation are also used for determining model fitness based on Table 4.

Table 4. Final model’s fitness indices

index	value	acceptable range	result
χ^2/df	0.79	<2	model confirmed
GFI	0.939	> 0.9	model confirmed
TLI	0.169	> 0.9	model confirmed
IFI	1.104	> 0.9	model confirmed
CFI	1.000	> 0.9	model confirmed
RMSEA	0.000	< 0.9	model confirmed

As it is seen in the table above, fitness indexes show that this model the best describes the type of data and the final model is fitted very well.

Research questions

1. Does the knowledge of concepts influence the knowledge of modeling?

To answer this question, path analysis was used. It is seen that the knowledge of concepts factor has a positive and direct effect on modeling ($p< 0.01$, t= 2.893, β = 0.246). The sum effect of this factor through other factors also has some effect on the knowledge of modeling ($p< 0.01$, t= 0. 643, β = 0.246). So the hypothesis is confirmed.

2. Does the knowledge of algorithm influence the knowledge of application?

To answer this question, path analysis was used. It is seen that the knowledge of algorithm has a positive and direct effect on knowledge of application ($p< 0.01$, t= 3.512, β= 0.299). The sum effect of this factor through other factors also has some effect on the knowledge of application ($p< 0.01$, t= 4. 720, β = 0.299). So the hypothesis is confirmed.

3. Does knowledge of application influence the knowledge of algorithm?

To answer this question, path analysis was used. It is seen that the knowledge of application has a positive and direct effect on knowledge of algorithm ($p < 0.01$, $t = 2.758$, $\beta = 0.231$). The sum effect of this factor through other factors also has some effect on the knowledge of algorithm ($p < 0.01$, $t = 2.857$, $\beta = 0.231$). So the hypothesis is confirmed.

4. Does knowledge of application influence the knowledge of theorems?

To answer this question, path analysis was used. It is seen that the knowledge of application has a positive and direct effect on knowledge of theorems ($p < 0.05$, $t = 2.103$, $\beta = 0.413$). The sum effect of this factor through other factors also has some effect on the knowledge of theorems ($p < 0.05$, $t = 2.039$, $\beta = 0.401$). So the hypothesis is confirmed.

3. Discussion and conclusion

Based on the results of the reported research studies, it can be said that conceptual and procedural knowledge are not learned independently. But there is a simultaneous or give and take relationship between them. In this process, sometimes, first the procedural knowledge is developed and brings about the development of conceptual knowledge, and some times vice versa. Learners' experience and characteristics, the topic which is taught and learning theories which lead the teaching process, are influential factors in this process.

In this research, based on the experimental model extracted, it can be concluded that in second order homogenous differential equations also the simultaneous or give and take relationship exists between these two types of knowledge in such a way that, first the procedural knowledge (the knowledge related to the theorems, concepts and algorithms for solving second order homogenous differential equations) develops, then the procedural knowledge bring about the development of conceptual knowledge (modeling physic problems with the help of second order homogenous differential equations and using first order differential equations for solving some second order differential equations). Finally, again they return to procedural knowledge and these topic concepts are reconsidered in order to perceive the relationship between them in the form of an orderly network.

The findings of Afamasaga (2004) and Waddy, Kim and Glass (2009) about conceptual learning of differential equation confirm the finding of the present study. This shows that students use their procedural knowledge for developing conceptual knowledge and then by returning to procedural knowledge make a relationship between these two types of knowledge, and in so doing, they deepen their knowledge. The problem which is emphasized in all these studies is trying to improve the students' ability in developing a true relationship between these two types of knowledge and gain a deep knowledge of the concepts when the knowledge is perceived truly and the concepts are in a rich network of relationship. Their transforming ability improves significantly and the new knowledge is more easily coordinated with existing structure which makes learning easy (Schoenfeld, 1992).

Examining the performance of the students who have participated in this research, shows that most of them faced some difficulties in solving modeling physical phenomenon. It seems it is because of over – emphasis on procedures and algorithms and not paying attention to conceptual learning. Trying to design more tasks in using and developing concepts and mathematics modeling and challenging activities in the teaching process, improves the relationship between conceptual and procedural knowledge of the students and, as a result, improves their learning and somewhat eliminates the learning difficulties.

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